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STABILITY OF THIN-SHELL WORMHOLES WITH MASSLESS EDDINGTON-BORN-INFELD INTERIOR AND SCHWARZSCHILD EXTERIOR

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Asymptotically flat, massless wormholes, like the Ellis-Bronnikov solution, are astrophysically interesting but individually unstable. This work investigates the stabilization of such wormholes by constructing a composite model where a massless interior is joined to a massive exterior spacetime across a spherical thin-shell. Specifically, we study a thin-shell wormhole whose interior is a massless Eddington-inspired Born-Infeld (EiBI) wormhole – a generalization of the Ellis-Bronnikov solution – glued to an exterior Schwarzschild vacuum. The EiBI theory introduces an additional parameter κ, whose influence on stability is a primary focus. Using the Visser "cut-and-paste" technique and the Darmois-Israel formalism, we derive the surface stresses of the thin shell composed of exotic matter. The stability of this Schwarzschild-EiBI wormhole under linearized radial perturbations is then analyzed via the García-Lobo-Visser method, which employs constraints on the thin-shell's effective "mass" and external "forces." Our detailed analysis reveals that the "external force" constraint is more restrictive than the "mass" constraint. The stability region of the glued wormhole is found to depend critically on the EiBI parameter κ: smaller values ($\kappa \sim 0.1 r_0^2$) yield broader stability regions compared to larger values ($\kappa \sim 2 r_0^2$), although exotic matter remains a necessity. Furthermore, increasing κ reduces the stability region and enlarges the required throat radius for gluing. The study demonstrates how the parameter κ, signifying a deviation from general relativity, can achieve the stability and geometry of obtained thin-shell wormholes.

Keywords: wormhole, thin shell, Eddington-Born-Infeld theory, stability.

Introduction. This analysis addresses a central theoretical challenge in wormhole physics: the inherent instability of traversable, asymptotically flat, zero Arnowitt-Deser-Misner (ADM) mass wormholes, despite their significant potential for astrophysical applications such as explaining anomalous microlensing events [1]. The study proposes and rigorously analyzes a stabilization mechanism wherein such a massless wormhole is not considered in isolation but as the interior core of a composite spacetime structure. This is achieved by surgically grafting, or "gluing," the massless interior geometry to an observationally relevant massive exterior spacetime across a spherically symmetric thin-shell interface, forming a complete traversable wormhole with a non-zero total ADM mass. The specific model constructed and analyzed here features an interior derived from Eddington-inspired Born–Infeld (EiBI) gravity – a theoretically motivated modification of general relativity containing a fundamental parameter κ with dimensions inverse to the cosmological constant. This EiBI wormhole [2] serves as a one-parameter generalization of the massless Ellis-Bronnikov solution, with $\kappa = 0$ recovering general relativity. The exterior is chosen as the standard Schwarzschild vacuum solution, representing a static, massive object.

Recent research on thin-shell wormholes has significantly evolved from their foundational Morris-Thorne-inspired constructs, focusing on mitigating the exotic matter problem and enhancing dynamical stability. A dominant trend involves sourcing the shell's matter through quantum field

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theoretical effects rather than ad-hoc exotic fluids. For instance, in [3] was proposed interpreting the shell as a manifestation of vacuum polarization (Casimir energy), offering a more physically plausible mechanism rooted in known phenomena. Concurrently, sophisticated stability analyses under various equations of state have become standard, as seen in the work of [4], who provided a comprehensive framework for linear stability of spherically symmetric thin-shell wormholes. Furthermore, the formalism has been actively extended beyond General Relativity. Studies in modified gravity theories, such as Einstein-Gauss-Bonnet, demonstrate how higher-order curvature terms can alter stability regions and energy conditions, as explored by [5]. This collective theoretical progress aims to anchor such wormholes within a more rigorous framework of quantum gravity and extended gravitational theories. Recent works have witnessed a pivotal shift toward the phenomenology of thin-shell wormholes, driven by the era of precision astrophysics. A major research thrust is to distinguish such objects from black holes via their observational imprints. Detailed simulations of gravitational lensing, including Einstein ring and photon ring substructures, suggest potentially discernible differences. Wang et al. [6] investigated these subtle distinctions in lensing systems, while Wu et al. [7] focused on the characteristics of shadows cast by thin-shell wormholess, comparing them with Event Horizon Telescope observations. This direct comparison with black hole candidates is crucial for formulating testable predictions. Additionally, thin-shell models are increasingly built from regular black hole metrics (e.g., [8]), conceptually linking different classes of exotic compact objects and exploring their stability under perturbation. The linear stability of thin-shell wormholes connecting a Schwarzschild black hole with different solutions was analyzed in [9–11].

The construction of thin-shell wormholes is formally executed using Visser's well-established "cut-and-paste" differential geometry technique [12]. Two distinct manifolds – the exterior Schwarzschild and the interior EiBI spacetime – are joined at a chosen hypersurface, the wormhole throat. Following the Darmois-Israel formalism [13], this gluing inevitably generates a thin layer of matter at the throat, quantified by surface energy densities and tensions. This matter violates the standard energy conditions, classifying it as "exotic," and is responsible for sustaining the wormhole throat against gravitational collapse by providing a necessary repulsive gravitational effect. The primary focus of the paper is to determine the conditions under which obtained thin-shell wormhole remains stable when subjected to small, spherically symmetric perturbations of the throat radius.

The stability analysis is performed using the innovative framework developed by García, Lobo, and Visser [14]. This method characterizes the dynamics of the thin shell not just by an equation of state, but by imposing two independent physical constraints: one on the perturbed "mass" of the thin shell itself, and a more restrictive one on the allowable "external forces" (or pressure anisotropies) that may act upon it. By linearizing the equation of motion around the equilibrium throat radius, the analysis derives the precise condition for stability in terms of the fundamental parameters of the system: the Schwarzschild mass, the equilibrium throat radius, and the EiBI parameter κ.

The detailed results demonstrate that the influence of the parameter κ is critical for the wormhole's stable configurations. An analysis reveals that smaller values of κ (on the order of $\kappa \sim 0.1 r_0^2$, where r_0 is a characteristic length scale) lead to substantially broader regions of stability in parameter space compared to the limiting Ellis-Bronnikov case or to models with larger κ values (e.g., $\kappa \sim 2 r_0^2$). This indicates that the EiBI theoretical framework, through κ, offers a tunable degree of freedom that can be optimized to enhance the mechanical stability of the thin-shell wormhole. Furthermore, the study finds that increasing κ not only reduces the stability region but also requires a larger equilibrium throat radius for the gluing process, as confirmed by analyzed embedding diagrams of the geometry into Euclidean space. Crucially, while the κ parameter can modulate the stability domain, the need for exotic matter at the throat remains an inescapable requirement of the model within this classical framework. This work thus provides a quantified examination of how modified gravity parameters can be leveraged to design theoretically more robust wormhole geometries that combine a massless, possibly phantom-free core with an externally observable mass, bridging a gap between theoretical constructs and potential phenomenological signatures.

EiBI solution. The action in the EiBI theory is given in the form [2]

$$S = \frac{2}{16\pi\kappa}\int d^4x\left(\sqrt{-\left|g_{\mu\nu}+\kappa R_{\mu\nu}(\Gamma)\right|}-\sqrt{-g}\right)+$$
$$+S_M[g,\Psi_M], \qquad (1)$$

where κ is the parameter with the inverse dimension of the cosmological constant Λ, $g_{\mu\nu}$ is the physical metric tensor, g is the determinant of the metric tensor, $R_{\mu\nu}$ is the symmetric part of the Ricci tensor constructed exclusively from the connection of the Christoffel symbols Γ, and S_M is the matter action, which depends only on the metric $g_{\mu\nu}$ and the matter field Ψ_M. To simplify the notation, the determinant of the tensor $g_{\mu\nu} + 8\pi\kappa R_{\mu\nu}$ is denoted in this article as $\left|g_{\mu\nu}+\kappa R_{\mu\nu}\right|$.

In the EiBI theory, the metric $g_{\mu\nu}$ and the connection Γ are considered as independent fields. By varying the action (1) with respect to the connection Γ and with respect to the real metric $g_{\mu\nu}$, we obtain the following field equations

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu}, \qquad (2)$$
$$q^{\mu\nu} = \tau(g^{\mu\nu} - 8\pi\kappa T^{\mu\nu}), \qquad (3)$$

where the auxiliary metric $q_{\mu\nu}$ is related to Γ by the relation

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}q^{\alpha\sigma}\left(\partial_\gamma q_{\sigma\beta} + \partial_\beta q_{\sigma\gamma} - \partial_\sigma q_{\beta\gamma}\right), \qquad (4)$$

$\tau = \sqrt{|g|/|q|}$ and the stress-energy tensor

$$T^{\mu\nu} = \frac{1}{\sqrt{-|g|}}\frac{\delta S_M}{\delta g_{\mu\nu}}. \qquad (5)$$

If the stress-energy tensor $T^{\mu\nu}$ vanishes in equation (3), then the real metric $g_{\mu\nu}$ is equal to the auxiliary metric $q_{\mu\nu}$ and EiBI gravity is completely equivalent to general relativity in a vacuum.

The exact solution for the EiBI wormhole, obtained by Harko et al. [2], is

$$ds^2 = -dt^2 + \left(\frac{1+2\kappa r_0^2/r^4}{1-r_0^2/r^2}\right)dr^2 + r^2 d\theta^2$$
$$+r^2\sin^2\theta\, d\phi^2. \qquad (6)$$

Metric (6) is a symmetric, twice asymptotically flat regular wormhole with asymptotic masses on both sides of the throat, where r_0 is the standard coordinate radius of the throat $r_{th} = r_0$, $r_0 \le r < +\infty$. In the limit $\kappa \to 0$, solution (6) reduces to the massless Ellis-Bronnikov wormhole of general relativity.

Embedding diagrams of the thin-shell Schwarzschild–EiBI wormhole. In this chapter, we will obtain and describe the geometry of a gluing between two manifolds, given a static radius a_0 of a Schwarzschild black hole on the outside and a massless EiBI wormhole on the inside. The static gluing radius will be chosen such that it is larger than the radius of the Schwarzschild black hole horizon, i.e., $r = a_0 > 2M$, on the one hand, and larger than the throat of the EiBI wormhole, i.e., $r = a_0 > b_0$, on the other. The regions $r \le 2M$ and b_0 are surgically removed from the corresponding manifolds.

Using Garcia-Lobo-Visser (GLV) method [14] we get redshift $\Phi_+ = 0$ and shape $b_+ = 2M$ functions for Schwarzschild black hole and redshift $\Phi_- = \frac{1}{2}\ln\left[\frac{r^4+2\kappa r_0^2}{r^2(r^2-r_0^2)}\right]$ and shape $b_- = \frac{r r_0^2(2\kappa+r^2)}{r^4+2\kappa r_0^2}$ functions for EiBI wormhole.

We will use the embedding diagrams to visualize the curvature of the thin-shell Schwarzschild–EiBI wormhole. The diagram will consist of three parts: the EiBI wormhole, the Schwarzschild black hole, and thin shell.

Let us consider the EiBI wormhole in three-dimensional space with the time coordinate t fixed in the equatorial plane, i.e. $\theta = \pi/2$. Then the line element (6) can be rewritten as

$$ds^2 = \left(\frac{1+2\kappa r_0^2/r^4}{1-r_0^2/r^2}\right)dr^2 + r^2 d\phi^2. \qquad (7)$$

Next, we construct the resulting slice in three-dimensional Euclidean space, defined by the cylindrical coordinates z, r and ϕ and has the form

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2. \qquad (8)$$

Let us rewrite equation (8) so that the embedded surface is described by the function $z = z(r)$

$$ds^2 = \left[1 + \left(\frac{dz}{dr}\right)^2\right]dr^2 + r^2 d\phi^2. \qquad (9)$$

Comparing equations (7) and (9), we obtain

$$\frac{dz^-}{dr} = -\frac{r_0}{r}\sqrt{\frac{r^2+2\kappa}{r^2-r_0^2}}. \qquad (10)$$

Since the EiBI wormhole is considered an internal region of spacetime, a negative root was chosen. After integration, we obtain the embedded surface function in the form

$$z^- = -\frac{i r_0 (r^2+2\kappa)^{\frac{3}{2}}}{6\kappa(r_0^2+2\kappa)^{\frac{1}{2}}}F_1\left(\frac{3}{2};\frac{1}{2},1;\frac{5}{2};\frac{r^2+2\kappa}{r_0^2+2\kappa},1+\frac{r^2}{2\kappa}\right), \qquad (11)$$

where $F_1(a;b_1,b_2;c;x,y)$ is the Appell hypergeometric function of two variables.

For the case of a Schwarzschild black hole, the embedded surface function is given by the formula [15]

$$z^+ = 2\sqrt{2M(r-2M)}. \qquad (12)$$

To obtain the embedding diagrams, it is necessary to rotate the functions of the embedded surfaces $z^{\pm}$ along the vertical z-axis. Figure 1 shows the embedding diagrams of a thin-shell wormhole obtained by gluing the outer Schwarzschild surface and the EiBI wormhole with the inner one for a static radius $a_0 = 1.5r_0$ for $\kappa = 0.1r_0^2$, for $a_0 = 2r_0$ for $\kappa = 0.5r_0^2$, for $a_0 = 2.5r_0$ for $\kappa = r_0^2$, for $a_0 = 3.5r_0$ for $\kappa = 2r_0^2$. When constructing the diagram, the relation $b_0 = 2M$ was used. It is evident from the figure that an increase in κ leads to a stronger curvature when passing from one space to another.

Analysis of the stability of the thin shell of the Schwarzschild–EiBI wormhole. To analyze the stability of a thin-shell Schwarzschild–EiBI wormhole, the GLV method [14] will be used. This method is based on estimating the mass of the thin shell and the effect of external forces on it under scalar perturbations.

Using the GLV method, we obtain constraints on the "mass" and "external force" for the thin-shell of the Schwarzschild–EiBI wormhole in the form

$$[m_s''(a_0)]''|_{a_0} \geq \frac{M}{r^3}\left(1-\frac{2M}{r}\right)^{-\frac{1}{2}}\left[\frac{M}{r}\left(1-\frac{2M}{r}\right)^{-1}+2\right] + \frac{4r_0^2 r^9 A}{(r^2-r_0^2)^{3/2}(r^4+2\kappa r_0^2)^{5/2}}, \quad (13)$$

$$[4\pi a\Xi(a)]''|_{a_0} \leq -\frac{3r^{14}r_0^2 B}{(r^2-r_0^2)^{5/2}(r^4+2\kappa r_0^2)^{7/2}}, \quad (14)$$

where

$$A = 3 + \frac{2(10\kappa - r_0^2)}{r^2} - \frac{36\kappa r_0^2}{r^4} + \frac{4\kappa r_0^2(5r_0^2-2\kappa)}{r^6} + \frac{12\kappa^2 r_0^4}{r^8},$$

$$B = 4 + \frac{5(8\kappa - r_0^2)}{r^2} + \frac{2r_0^2(r_0^2-64\kappa)}{r^4} + \frac{10\kappa r_0^2(15r_0^2-8\kappa)}{r^6} + \frac{8\kappa r_0^4(26\kappa-7r_0^2)}{r^8} - \frac{156\kappa^2 r_0^6}{r^{10}} + \frac{40\kappa^2 r_0^8}{r^{12}} + \frac{8\kappa^3 r_0^8}{r^{14}}.$$

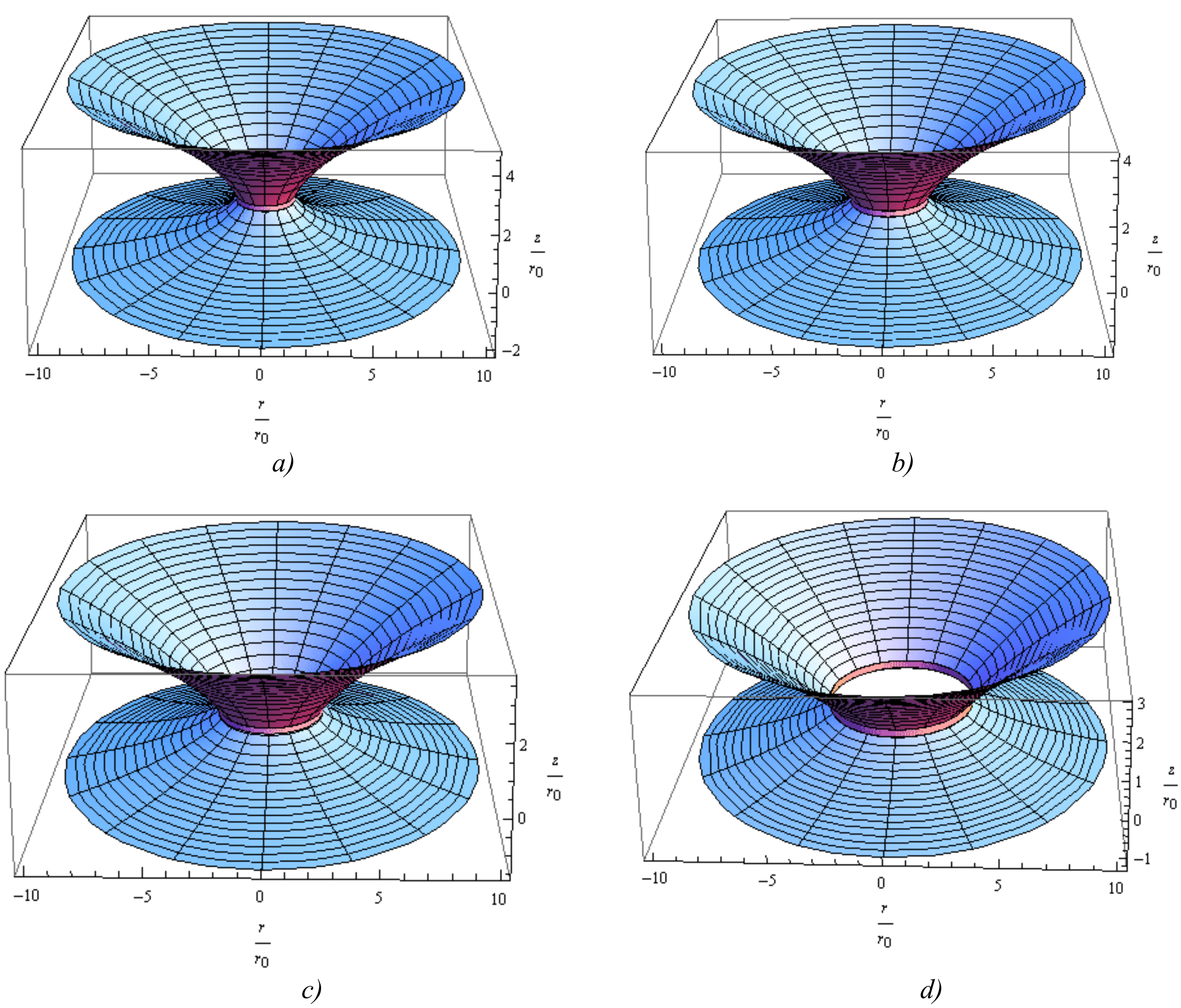

Fig. 1. Embedding diagrams of the thin-shell Schwarzschild–EiBI wormhole for different values of κ

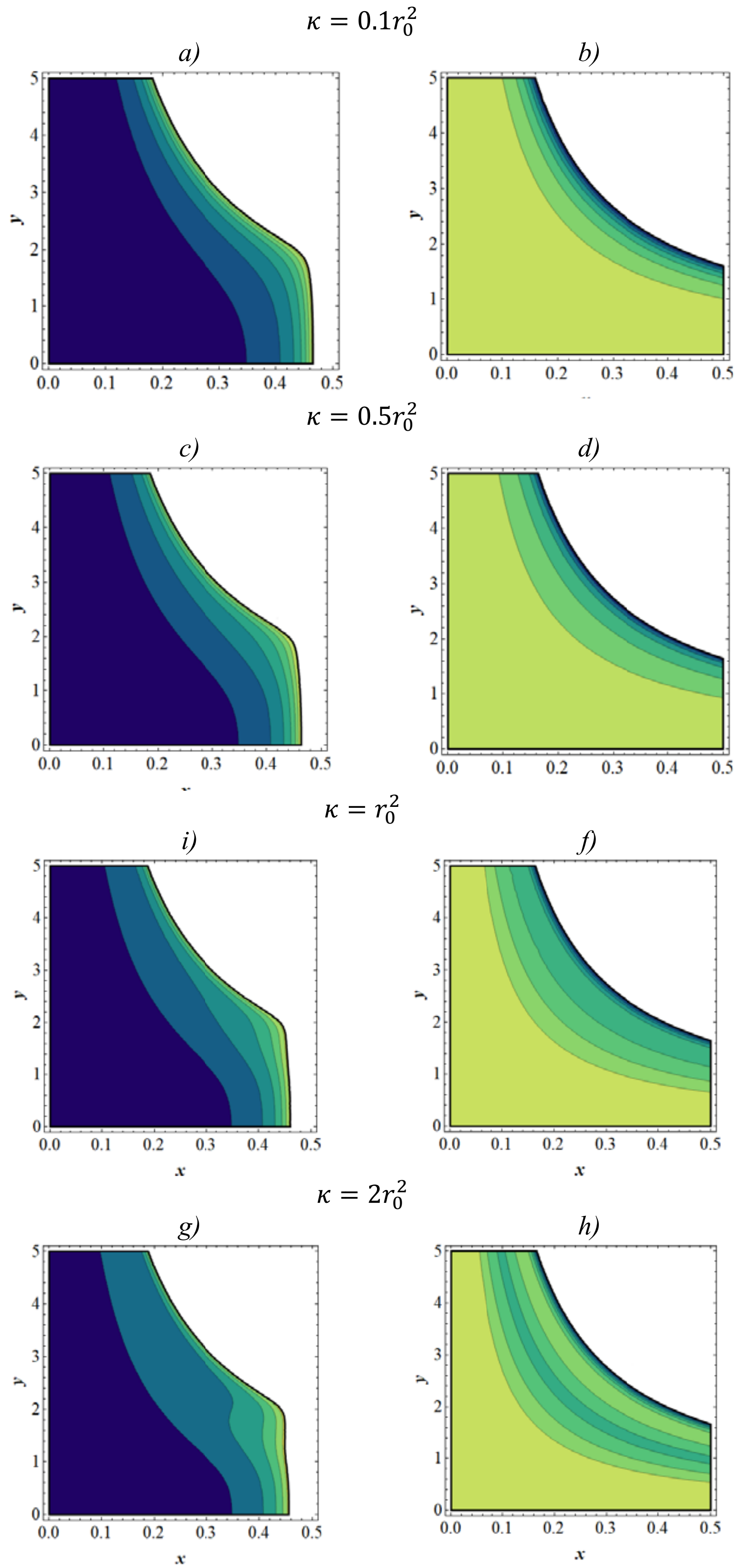

Fig. 2. Two-dimensional contour graphs of the constraint functions for "mass" and "external forces" for different values of the EiBI parameter κ

In Fig. 2, the stability region of the glued Schwarzschild–EiBI wormhole, obtained using the "mass" constraint, is located above the surface (13), and the stability region obtained using the "external forces" constraint is located above the surface (14). To construct the contour plots, the dimensionless quantities $x = M/a_0$ and $y = r_0/M$ were used. The thin-shell will be stable if both conditions are met. Fig. 2 *a, b* shows the case when the parameter $\kappa = 0.1r_0^2$. From Fig. 2 *a, b* it is evident that the thin shell is stable for $x \geq 0.181$ and $y \geq 1.62$, i.e. the gluing radius of the stable wormhole is determined in the range from $2M$ to $5.52M$, and the radius r_0 varies in the range from $1.62M$ to $5M$. An increase in the parameter κ leads to a decrease in the stability region of the Schwarzschild–EiBI wormhole, and for the case $\kappa = 2r_0^2$ the thin shell is stable at $x \geq 0.19$ and $y \geq 1.69$, i.e. the cross-linking radius of a stable wormhole is determined in the range from $2M$ to $5.26M$, and the radius r_0 varies in the range from $1.69M$ to $5M$.

Conclusions. The massless, traversable Ellis–Bronnikov wormhole, while of significant astrophysical interest for applications such as galactic microlensing, is known to be unstable in its isolated form. The central element of our investigation was the influence of the EiBI theory parameter, κ, which generalizes the Ellis–Bronnikov solution and serves as a crucial degree of freedom controlling the geometry of the interior. In this paper, we have demonstrated a viable stabilization mechanism by considering it not as an isolated object, but as the interior component of a composite spacetime structure. The key idea is that the stability of a zero-ADM-mass wormhole can be achieved by surgically grafting it, via the thin-shell formalism, onto a massive exterior spacetime in our case, the Schwarzschild black hole. The massless interior partner then inherits its stability from the dynamics of the entire glued construction, while preserving its asymptotic masslessness. This constitutes the core conceptual contribution of our work.

In this paper, we obtained a thin-shell wormhole model by gluing together an EiBI wormhole as interior and a Schwarzschild black hole as exterior at a given static radius a_0 using a "cut-and-paste" procedure. Gluing the two manifolds together results in a thin shell. Using the Garcia-Lobo-Visser formalism, we obtained a "mass" constraint on the thin shell and an "external force" constraint on the stability of the Schwarzschild–EiBI wormhole under linearized spherically symmetric perturbations. It is shown that the "external force" constraint is stronger than the "mass" constraint. The wormhole's stability region decreases with increasing EiBI parameter κ. We also constructed diagrams of the Schwarzschild–EiBI wormhole embedding in three-dimensional Euclidean space. Analysis of the diagrams showed that increasing the parameter κ leads to an increase in the wormhole stitching radius.

In summary, the Schwarzschild–EiBI thin-shell wormhole serves as a compelling example of how a classically unstable, massless wormhole solution can be stabilized as an interior partner in a composite structure. The present work systematically elucidates the stability criteria for such an object, demonstrating that stability is achievable within specific regions of the parameter space, crucially controlled by the EiBI parameter κ. While the requirement for exotic matter at the thin shell persists, our analysis quantifies how modified gravity parameters can be leveraged to design theoretically viable and stable wormhole geometries that bridge the gap between massless interiors and observationally relevant massive exteriors. The primary goal of this paper was to shed light on these stability aspects, and we conclude that the EiBI generalization offers a valuable pathway towards securing the stability of glued wormholes with massless cores.

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СТАБИЛЬНОСТЬ КРОТОВЫХ НОР С ТОНКОЙ ОБОЛОЧКОЙ ЭДДИНГТОНА-БОРНА-ИНФЕЛЬДА-ШВАРЦШИЛЬДА

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Асимптотически плоские кротовые норы с нулевой массой Арновитта–Дезера–Миснера (АДМ), подобные решению Эллиса-Бронникова, представляют астрофизический интерес, но по отдельности нестабильны. В данной работе исследуется стабилизация таких кротовых нор путем построения составной модели, в которой безмассовая внутренняя область соединена с массивным внешним пространством-временем через сферическую тонкую оболочку. В частности, мы изучаем кротовую нору с тонкой оболочки, внутренняя область которой представляет собой безмассовую кротовую нору Эддингтона–Борна–Инфельда (ЭБИ), – обобщение решения Эллиса-Бронникова – соединенную с внешним вакуумом Шварцшильда. Теория ЭБИ вводит дополнительный параметр κ, влияние которого на стабильность является основным предметом исследования. Используя метод «разрезания и склейки» Виссера и формализм Дармуа–Израэля, мы выводим поверхностные напряжения тонкой оболочки, состоящей из экзотической материи. Затем устойчивость этой кротовой норы Шварцшильда-ЭБИ под действием линеаризованных радиальных возмущений анализируется с помощью метода Гарсиа–Лобо–Виссера, который использует ограничения на эффективную «массу» тонкой оболочки и «внешние силы». Наш подробный анализ показывает, что ограничение на «внешнюю силу» является более строгим, чем ограничение на «массу». Обнаружено, что область устойчивости склеенной кротовой норы критически зависит от параметра ЭБИ κ: меньшие значения ($\kappa \sim 0.1 r_0^2$) дают более широкие области устойчивости по сравнению с большими значениями ($\kappa \sim 2 r_0^2$), хотя экзотическая материя остается необходимой. Кроме того, увеличение κ уменьшает область устойчивости и увеличивает требуемый радиус горловины для сшивки. Исследование демонстрирует, как параметр κ, обозначающий отклонение от общей теории относительности, может модулировать устойчивость и геометрию составных кротовых нор с тонкой оболочкой.

Ключевые слова: кротовая нора, тонкая оболочка, теория Эддингтона–Борна–Инфельда, стабильность.