КНИЖНОЕ ОБОЗРЕНИЕ

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Shamil U. Galiev (2020) Simulation of Extreme Waves in Technology and Nature.
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Volume 1. **The Evolution of Extreme Waves and Resonances.**

Galiev's book is devoted to waves and their mathematical modeling. These are problems that have been studied for a long time in science. Therefore, the place occupied by the book in modern science and the significance of its results will not be completely clear to all readers, if we, at the very beginning of the review, do not lead the readers very briefly into the history of the problem of waves and their modeling.

The formation of the concept of waves by mankind goes back to ancient times. However, this concept acquired a scientific meaning relatively recently, at the end of the Middle Ages, in the works of Leonardo da Vinci, Galileo, Descartes and Huygens. In particular, da Vinci came to the conclusion that the propagation of both sound and light has a wave nature. Galileo wrote … *That a glass of water may be made to emit a tone…. Sometimes happens, the tone of the glass jumps an octave higher…. each of the aforesaid waves divides in two….* Galileo also discussed a row parallel, equidistant streaks excited on a brass plate. Thus, Galileo was studying the wave patterns on water and solid body surfaces, and described the nonlinear wave effect. Pondering the problem of creating a pendulum clock, Galileo and Huygens approached the problem of resonances and synchronization. Later it was found that the vast majority of the observed waves can be described as harmonic oscillations, as shock waves, or as solitary waves (various types of solitons). These are the main classes of waves well studied by the end of the 20th century.

The idea of describing nature and modeling natural processes using numbers originated in even deeper antiquity. Pythagoras formulated it quite clearly. He approached this idea by studying the harmonics of string vibrations. He singled out his own (resonant) vibration modes (interestingly, Galileo's father repeated Pythagoras's experiments with stretched strings almost 2000 years later. So the interest in vibrations and waves in the Galilean family was almost hereditary!). The numbers allowed Pythagoras to formulate clearly his famous theorem. However, the idea of mathematical modeling of natural and technical processes received an almost modern sound only in the works of Galileo and Newton. However, they did not explicitly set the task of modeling wave phenomena. The first wave equation was derived in 1748 by d'Alembert (D'Alembert) [2]. It describes the vibrations of the string and many other wave processes and has the form

$$u_{tt} - c^2 u_{aa} = 0\,. \tag{1}$$

Perhaps, when deriving this equation, d'Alembert had in mind the results of Pythagoras, having received a very general result on which the entire modern wave theory is based. In particular, some ways of generalizing the d'Alembert equation are related to taking into account nonlinear effects. It is interesting that Galiev explicitly considers the theory of his book as a generalization of d'Alembert's result to the cases of strongly nonlinear waves, waves with complex folded (multivalued) profiles, and particle – waves. That is, he considers extreme waves that have practically not been studied in science. Modeling these waves is reduced to using one of the following equations

$$\begin{aligned} & u_{tt} - ghu_{aa} = g(2h_a u_a + h_{aa}u) - \\ & -3gh(1-2u_a)u_a u_{aa} + h_{aa}u) + \\ & +(\tfrac{1}{3}h^2 - \sigma\rho^{-1}g^{-1})u_{ttaa} + \\ & +2\bar{\mu}\rho^{-1}h^{-1}(2hu_{taa} + 2h_a u_{at} + h_{aa}u_t) - \rho^{-1}p_{0a} + \\ & +\rho^{-1}h^{-1}p_{31}(a,t,c=0) - \rho^{-1}h^{-1}p_{31}(a,t,c=h), \end{aligned} \tag{2}$$

$$\begin{aligned} & \Phi_{tt} - gh\Phi_{aa} = hk^{-1}\Phi_{ttaa} + \tfrac{1}{2}hk^2(\Phi_{tt}\Phi)_a - \\ & -\tfrac{1}{3}hk^3[\Phi\int (\Phi_a{}^2)_{tt}\,da + 2\Phi_{tt}\int \Phi_a{}^2 da]_a + \\ & +\rho^{-1}p_{31}(a,t,c=0) + \rho^{-1}hp_{0,a}, \end{aligned} \tag{3}$$

$$\phi_{tt} - c_0^2\phi_{aa} = -m^2\phi + \lambda\phi^3. \tag{4}$$

The above equations cover a huge generality of strongly nonlinear one-dimensional waves. In particular, equation (2) describes the propagation of extreme waves in different media. In this book, (2) is used to simulate waves in gas and on the surface of shallow fluids. Equation (3) is written for ocean waves, and nonlinear Klein-Gordon equation (4) is derived for waves in strong scalar fields.

The left sides of equations (2)–(4) coincide with the d'Alembert equation (1). The right sides differ from (1) mainly in the presence of nonlinear terms. Thus, equations (2)–(4) are a generalization of (1). Using this, the author constructs approximate solutions of the indicated equations of the form of traveling waves introduced by d'Alembert. These solutions claim to be an original and unified description of the vast generality of nonlinear wave processes.

Jean Leron d'Alembert

This concludes a very short introduction to the book review. We emphasize that the purpose of this review is by no means a thorough review of the content of the book, criticism or approval of its results. Our goal is to give the reader an idea of the problems considered in it, the results obtained, and their connection with other results of well-known researchers. In particular, following them the author tries to use the most simple mathematical apparatus, which at the same time is a significant development of ideas, models and mathematical research of such geniuses as Euler, d'Alembert, Laplace, Faraday and Darwin [2–7] and of such wonderful creators as Green, Airy, Boussinesq, Kirchhoff and others [8–14]. It is on the basis of this approach that the main results of the book are obtained. At the same time, as the reader can easily see, the book in question is a natural development of ideas and results presented in more modern studies [14–31].

The book (volume I) consists of 4 parts and 15 chapters.

Part I contains the basic equations and some results that illustrate the main ideas and objectives of the book. In the first chapter, equations of the form (2) and (3) are derived. Special attention is paid to long waves (2). It is shown that from this equation follow the equations obtained by Airy and Boussinesq (George Biddell Airy and Joseph Valentin Boussinesq) [10–11].

George Biddell Airy

Chapter 2 shows the possibility of the existence of multivalued waves whose profile is well described by the so-called Euler figures (Fig. 1). Such waves are described by equations (2) and (4). It is emphasized and demonstrated that in some limiting cases, equations (2) and (4) can be transformed into each other, as well as into one-dimensional equations of electrodynamics, the Schrödinger and Gross-Pitaevsky equations. Consequently, all the above equations in some cases have solutions describing some of Euler's figures [20–23]. The author concludes that these figures can determine the waves arising in various structures, starting from the atomic level and then through the waves of the oceanic scale to the waves that existed or exist in the Universe. Of course, the appearance of such extreme waves when there is the gravity (that is, under terrestrial conditions) is the exception rather than the rule!

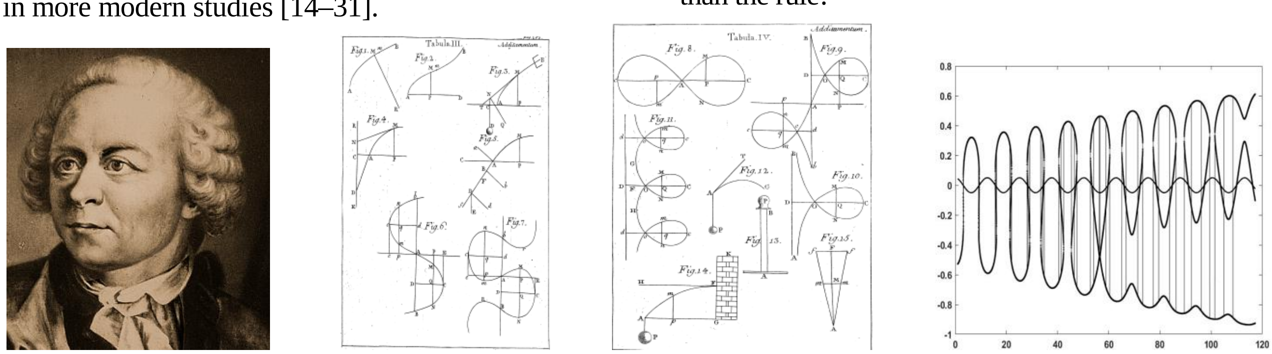

Fig. 1. Leonhard Euler, examples of Euler's figures (center) and an example of transresonant evolution of an extreme wave from an almost stepwise to a multi-valued wave and then to the particle – waves that is further transformed into cnoidal and harmonic waves (right)

Thus, in Part I the fundamental result is formulated. It turns out that Euler's solutions obtained in 1744 describe not only some shapes of rods and strings [3, 12, 13], but also some extreme wave profiles. It is underlined, that Galiev, based largely on Euler's results, predicts the existence of waves that have not yet been practically observed. This situation is the opposite of the case of the discovery of a soliton, which was first observed by Russell Scott [9], and only 30 years later was mathematically described by Boussinesq.

In Parts II and III, equations (2) and (3) are used to investigate the extreme waves generated in finite pipes, containers, seas and oceans. The possibility of these waves appearing is associated with the phenomenon of resonance. Let us illustrate it with examples of waves excited in a closed tube by an oscillating piston (Fig. 2 on the left) and on the surface of a liquid excited in a container (Fig. 2 center and right).

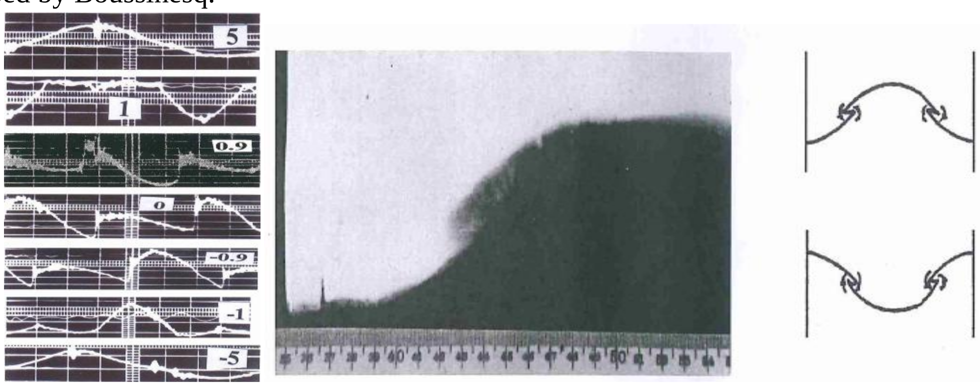

Fig. 2. Results of the experiments of Ilgamov M.A. and Sadykov A.V. (left) [16] (see, also, Fig. 3). Multi-valued resonant waves on the liquid surface (center and right) [24]. The folds on the profile of multivalued waves correspond to the curve Fig. 7 calculated by Euler (Fig. 1)

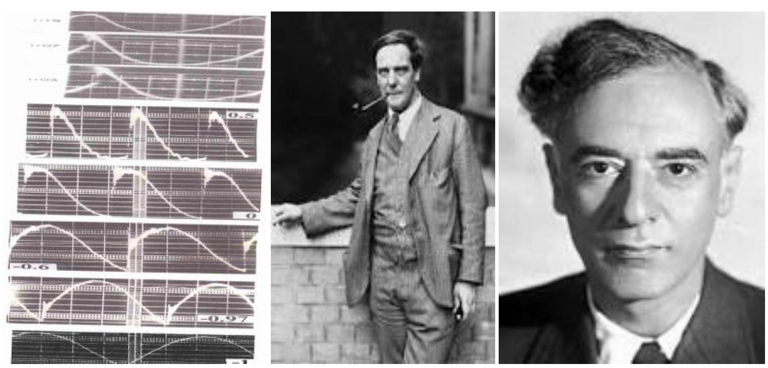

Fig. 3. Experimental results of M.A. Ilgamov and A.V. Sadykov (left) [16]. Peter Kapitsa (center) and Lev Landau (right)

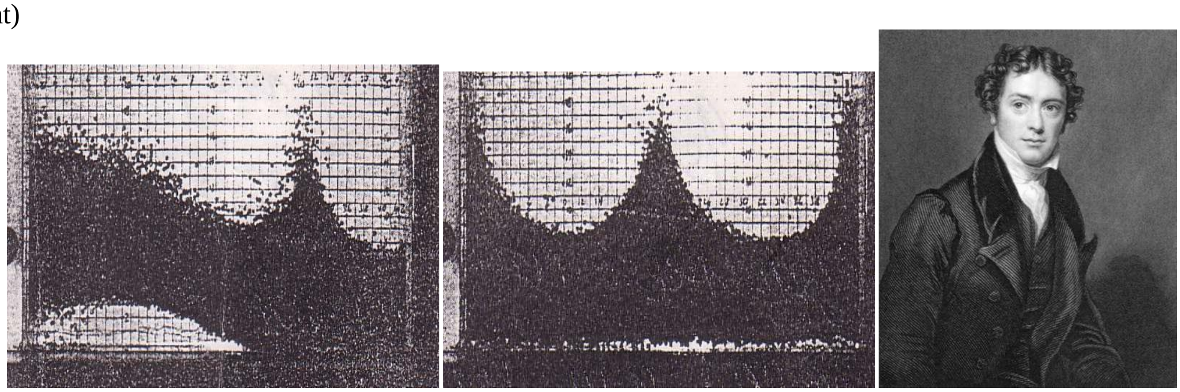

Fig. 4. Michael Faraday and extreme Faraday waves.

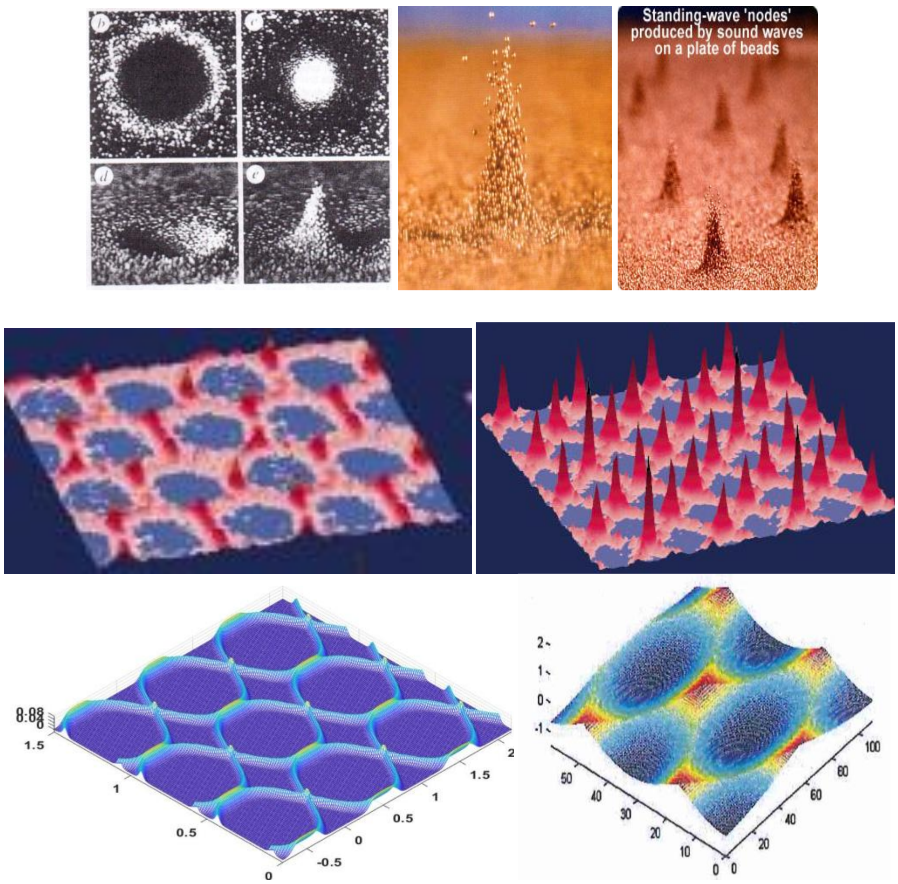

Fig. 5. Periodical column and crater (oscillons) on the surface of a vertically-excited layer

Note that experiments with surface waves are of great interest for theoretical and experimental physics [25–34]. The results of experiments with shock waves in tubes are also still of considerable applied interest. They are related to the optimization of rocket engines. In the Soviet Union, these waves began to be experimentally studied many years ago due to the initiative of the Nobel Prize winners Pyotr Kapitsa and Lev Landau (Fig. 3).

Part II is devoted to the construction of a generalized d'Alembert solution for an equation of the form (2) and the use of this solution in the study of resonant oscillations of media in finite resonators. First, in Chapter 3, generalized solutions of an equation of the form (2) are constructed. Further in Chapter 4 of this part, quadratic – nonlinear, viscous, dispersion and frequency effects on extreme waves excited in closed resonators are studied. Solutions describing resonant extreme shock waves and solitons are constructed. In particular, the data of the experiments of Ilgamov M.A. and Sadykov A.V. are modeled (Figs. 2 and 3) [16]. Chapter 5 covers both closed and half-open resonators. Attention is focused on the influence of cubic nonlinearity and the possibility of the appearance in the resonators of waves corresponding to the Euler figures, as well as on the separation of liquid droplets from its free surface. This takes place during atomization of a liquid surface at resonance of its volume. The multivalued waves shown in Fig. 2 accompanied by the appearance of the particle-waves (Fig. 1, right). A number of results in Chapters 4 and 5 are extended in Chapter 6 to cases of resonant forced spherical waves.

Chapter 7 is devoted to parametrically excited extreme Faraday waves.

Michael Faraday studied the effect of vertical vibrations on a layer of loosely bound materials [5]. He found the formation of small hills on the initially smooth surface of the layers and the slow convection of particles due to vibrations. Perhaps his most popular discovery in the summer of 1831 was that surface waves in a vertically oscillating liquid oscillate at exactly half the frequency of exposure. Rayleigh [30] recognized that the waves were the result of parametric resonance. For an inviscid fluid, this idea can be transformed into the Mathieu equation [31]. Recent experimental studies of granular media excited by vertical vibrations have demonstrated a wide variety of nonlinear wave phenomena.

Experimental and theoretical studies of a vertically oscillating low-viscosity liquid have demonstrated the emergence of many regular structures (surface patterns, for example, rolls, squares or hexagons). Some of them are shown in Fig. 5 (upper and middle rows – experiment, lower row – calculations). The theory of these localized periodic waves, which are called ossilons, was developed in [32]. The richness of surface patterns increases in the case of a highly viscous liquid [31]. In particular, when accelerating in excess of *g*, the effective gravity becomes negative and the layer loses contact with the base. A gap forms between the vibrating base and the layer. After the impact collision, the gap disappears until the next cycle, where the process is repeated (Fig. 4, center and right).

Thus, Chapter 7 touches upon a range of issues related to the excitation of extreme Faraday waves. Under some conditions, the instability of these waves generates vortices and turbulence [32–35].

Traditional methods of studying parametrically excited waves in above cases do not give a satisfactory result. Therefore, Galiev is constructing his own approach to the study of extreme parametrically excited surface waves. This approach has made it possible to describe a huge amount of experimental data published in the last 20 years. Galiev connects these data and the results of his theory with the effects that occur during earthquakes and seaquakes observed by Charles Darwin, approximately at the same time (1835), when Faraday conducted his experiments.

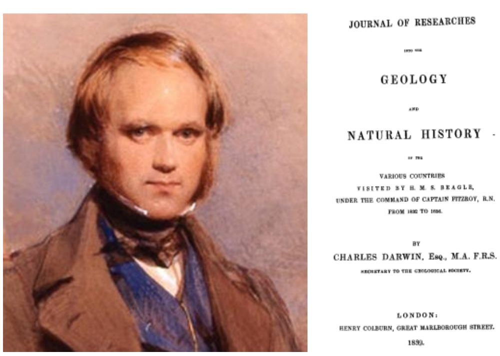

Fig. 6. Charles Robert Darwin and the cover of the first edition of his book.

The third part of the book deals with ocean waves. Equation (2) and (3) are used. In Chapter 8, an approximate solution to Eq. (2) is constructed, taking into account the variability of depth, quadratically – nonlinear effect and friction of the fluid against the bottom. Thus, a generalization of the Green law obtained in 1837 is given (George Green is the famous English mathematician, physicist and mechanic) [8]. Further, this law is used to study the resonance distortion and amplification of ocean waves over underwater topographies and when running ashore. As a result, extreme ocean waves sometimes occur. These include tsunamis (chapter 9) and giant storm waves (chapter 10).

At the first these phenomena were described and explained by Charles Darwin in his wonderful book "TheVoyage of the Beagle" [6] (Fig. 6). Darwin emphasized the dependence of the amplitude and shape of the tsunami on the coast and changes in depth. He writes: *...and lastly, of its size being modified (as appears to be the case) by the form of the neighbouring coast* .... Chapter 9 also discusses the coastal evolution of ocean waves into shock forms and breakers. Stormy extreme ocean waves are different from tsunamis. Tsunamis become dangerous only when they reach the coastal zone. In contrast, extreme ocean waves most often occur far offshore. The Beagle met such a wave off Cape Horn. Charles Darwin wrote ... *At noon a great sea broke over us, ... The poor Beagle trembled at the shock, and for a few minutes would not obey her helm;... Had another sea followed the first, our fate would have been decided soon and for ever* **..**. Chapter 10 shows that the occurrence of such extreme ocean waves can be associated with wave resonance due to underwater topographies and inhomogeneities in ocean thickness due to changes in density and temperature or the presence of marine organisms.

Chapter 11 explores wind waves, specifically wind-wave resonance. The one-dimensional wave theory developed in Chapter 1 is used. It is known that the wind acts on the ocean surface in different ways. First, by the surface friction. This mechanism generates ripples and waves of moderate amplitude. As the waves grow, their effect on the air flow increases. The wind action begins to depend on the inclination of the wave surface element to the direction of the air flow (wind). In this chapter, an approximate theory is developed that takes into account these factors in cases when the speed of the wind and waves are close or equal to each other. Examples of model calculations are given. In the last chapter of Part III, a model of the transresonant evolution of harmonic waves into vortices is considered (Fig. 7).

This evolution includes the appearance of high harmonics and then extreme waves and vortices. On the whole, apparently, Chapter 15 only outlines the direction of research, which, in the author's opinion, opens up a new way of describing the occurrence of wave turbulence [32–35].

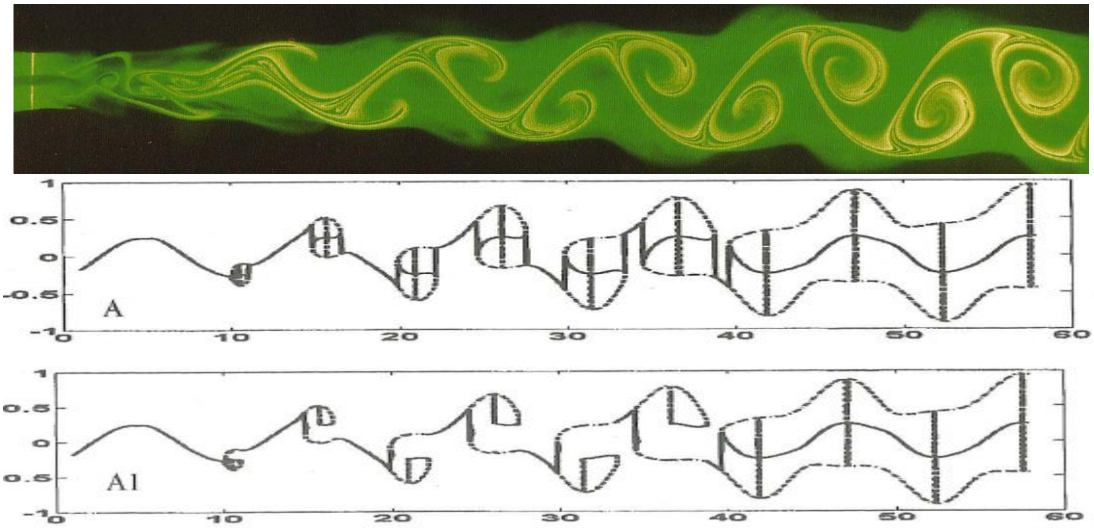

Fig. 7. Experimental data and an example of calculating wave evolution into vortices

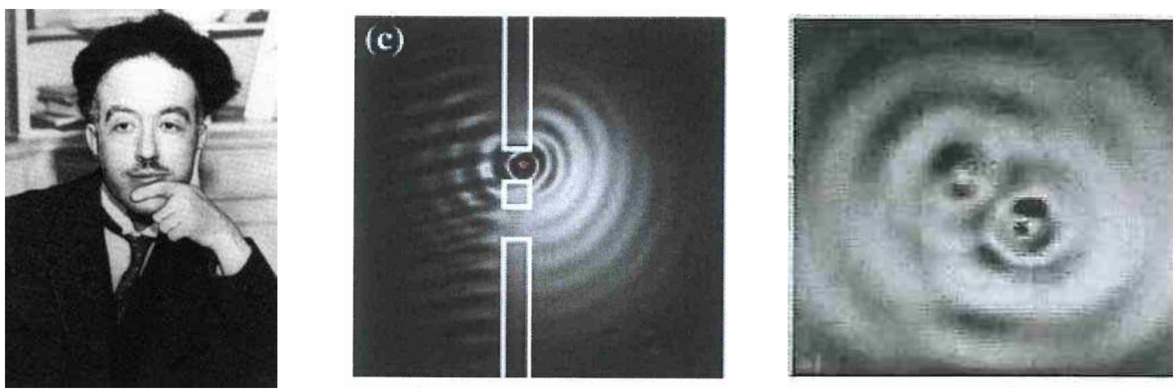

Fig. 8. De Broglie (Louis de Broglie) and the particle – waves. The passage of a particle – waves through a double slit (center) and the interaction of two particle – waves (right)

In Part IV, previously obtained results are extended to physical fields. Fields are a very specific and complex area of science, for which there are no fundamental experiments that determine the possibility of extreme waves. It is emphasized that equations (2), (4), the equations of electrodynamics, Schrödinger and Gross-Pitaevsky follow from each other and therefore have similar solutions [23]. Therefore, in this part, the author makes extensive use of the experience gained in the study of extreme surface waves. At the same time, the studies and results of Part IV are enough far from analogous to those obtained for surface waves. The fact is that the study of scalar fields opens the way to a certain understanding of the fundamental problems of the origin of matter and even the Universe. The peculiarity is also in the fact that in relation to the indicated regions the solutions should describe the particle-waves and answer the questions of quantum mechanics, and some of them do not yet have a generally accepted explanation.

This idea is illustrated in Chapter 13, where the solutions of the nonlinear Klein-Gordon equation describing Euler's figures, as well as the particle-waves moving in space, are constructed. It is interesting that such particle-waves arise in some cases of vertical excitation of liquid layers. These are drops on the layer surface that can move or "walk" together with the wave [25–28].

Particles (drops) located at the top of a traveling surface wave have attracted such attention because they can be interpreted as a kind of analogue of quantum waves. Such a particle-wave can be considered also as some amazing variant of Faraday waves, which at the same time can be considered as some analogue of pilot de Broglie waves (Fig. 8).

These wave – particles are used to explain the results of classic double slit experiments. The history of these experiments goes back more than two centuries. Their results played a significant role in the discovery of the wave nature of light, and then formed the basis for the most important provisions of quantum mechanics. However, these results are difficult to interpret unambiguously. This is especially difficult to do in the light of recent experiments. The purpose of Chapter 14 is to analyze the results of these experiments and offer their new understanding, based on taking into account the possibility of the appearance of virtual particle-waves, the theory of which was developed in Chapter 13, during implementation of the experiments.

By themselves, the scalar fields described by the Klein-Gordon equation are not "made" of anything. Fields are what the world is made of. Therefore, fields are often the simplest way to describe various natural phenomena. In particular, field theories are often used to introduce new concepts and methods. In Chapter 15, scalar fields and their approximate solutions in the form of extreme particle-waves are used to simulate quantum processes.

Conclusion on the review. Extreme waves are relatively poorly studied objects about which they often say “Extreme waves that appear from nowhere and disappear without a trace anywhere” [36 – 38]. In this regard, their behavior resembles the situation with elementary (quantum) particles, which also unexpectedly appear, move along unknown trajectories and disappear into no one knows where if the measuring device does not have time to fix them. The main goal of the book is to study these amazing waves using solutions to nonlinear wave equations and experimental data.

The main concept used in this is resonance. It is impossible to name a field of knowledge in which this concept could not be used. However, although the phenomenon of resonance is well known, at the same time, the evolution of waves near resonances has been insufficiently studied. Indeed, in resonance regions, nonlinear phenomena begin to play an important role; therefore, certain difficulties arise in the study and solution of nonlinear wave equations.

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